Areas of Triangles and Quadrilaterals (pp 372-376)

AREA POSTULATES

POSTULATE 22 ***Area of a Square Postulate***

The area of a square is the square of the length of its side, or $A = s^2$.

POSTULATE 23 ***Area Congruence Postulate***

If two polygons are congruent, then they have the same area.

POSTULATE 24 ***Area Addition Postulate***

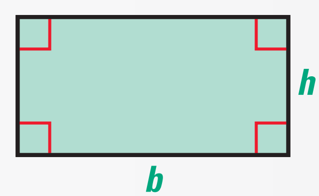
The area of a region is the sum of the areas of its nonoverlapping parts.

AREA THEOREMS

THEOREM 6.20 ***Area of a Rectangle***

The area of a rectangle is the product of its base and height.

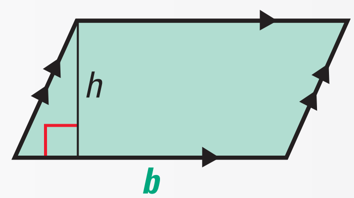
$$A = bh$$



THEOREM 6.21 ***Area of a Parallelogram***

The area of a parallelogram is the product of a base and its corresponding height.

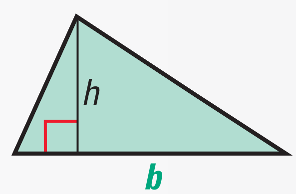
$$A = bh$$



THEOREM 6.22 ***Area of a Triangle***

The area of a triangle is one half the product of a base and its corresponding height.

$$A = \frac{1}{2}bh$$

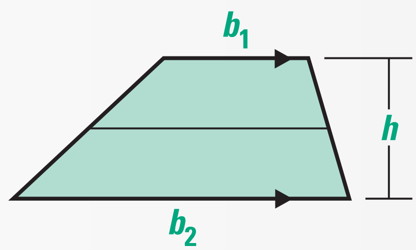


THEOREMS

THEOREM 6.23 ***Area of a Trapezoid***

The area of a trapezoid is one half the product of the height and the sum of the bases.

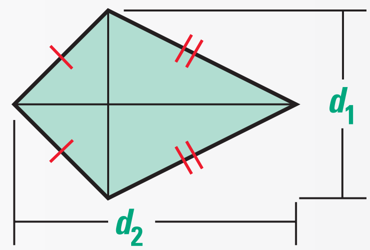
$$A = \frac{1}{2}h(b_1 + b_2)$$



THEOREM 6.24 ***Area of a Kite***

The area of a kite is one half the product of the lengths of its diagonals.

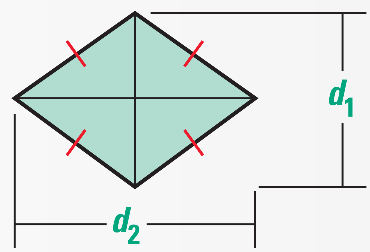
$$A = \frac{1}{2}d_1d_2$$



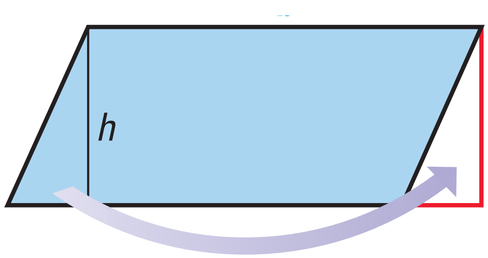
THEOREM 6.25 ***Area of a Rhombus***

The area of a rhombus is equal to one half the product of the lengths of the diagonals.

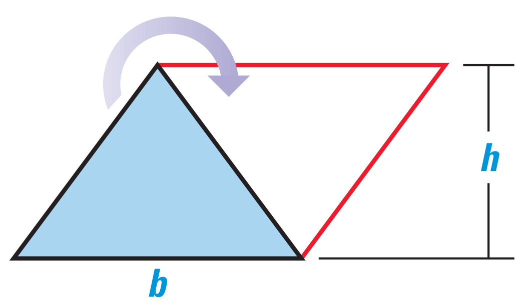
$$A = \frac{1}{2}d_1d_2$$



You can justify the area formulas for triangles and parallelograms as follows.

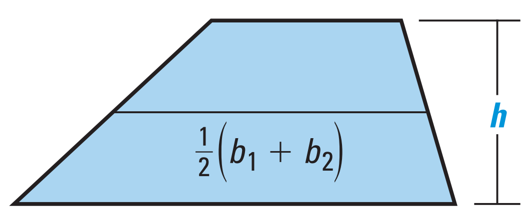


The area of a parallelogram is the area of a rectangle with the same base and height.



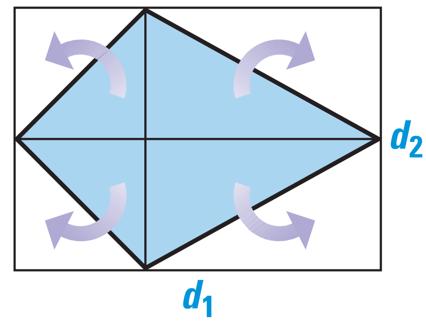
The area of a triangle is half the area of a parallelogram with the same base and height.

Area = **Length of Midsegment** • **Height**



The diagram at the right justifies the formulas for the areas of kites and rhombuses.

The diagram shows that the area of a kite is half the area of the rectangle whose length and width are the lengths of the diagonals of the kite. The same is true for a rhombus.



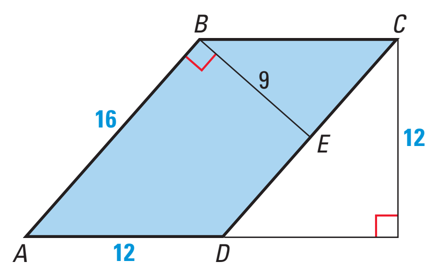
$$A = \frac{1}{2}d_1 d_2$$

STUDENT HELP

Study Tip
To find the area of a parallelogram or triangle, you can use any side as the base. But be sure you measure the height of an altitude that is perpendicular to the base you have chosen.

EXAMPLE 1 *Using the Area Theorems*

Find the area of $\square ABCD$.



SOLUTION

Method 1 Use $\overline{AB}$ as the base. So, $b = 16$ and $h = 9$.

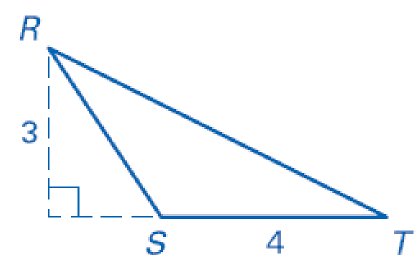
Area $= bh = 16(9) = 144$ square units.

Method 2 Use $\overline{AD}$ as the base. So, $b = 12$ and $h = 12$.

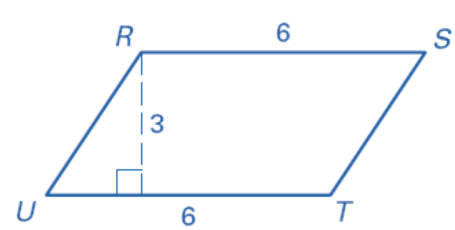
Area $= bh = 12(12) = 144$ square units.

Notice that you get the same area with either base.

1. Example: Find the area of $\triangle RST$.



2. Find the area of $\square RSTU$.



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EXAMPLE 2 *Finding the Height of a Triangle*

Using Algebra

Rewrite the formula for the area of a triangle in terms of h. Then use your formula to find the height of a triangle that has an area of 12 and a base length of 6.

SOLUTION

Rewrite the area formula so h is alone on one side of the equation.

$A = \frac{1}{2}bh$ **Formula for the area of a triangle**

$2A = bh$ **Multiply both sides by 2.**

$\frac{2A}{b} = h$ **Divide both sides by *b*.**

Substitute 12 for A and 6 for b to find the height of the triangle.

$$h = \frac{2A}{b} = \frac{2(12)}{6} = 4$$

▶ The height of the triangle is 4.

3. Rewrite the formula for the area of a rectangle in terms of b. Then use your formula to find the base of a triangle that has an area of 48 and a height of 3.

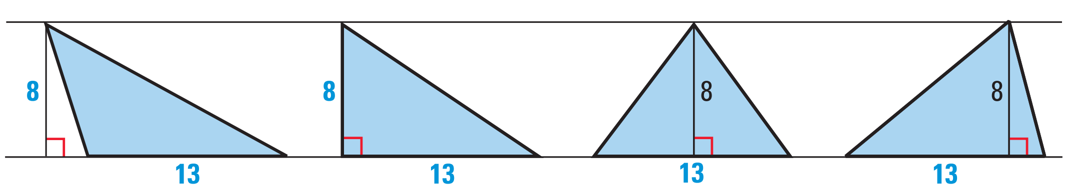
EXAMPLE 3 *Finding the Height of a Triangle*

A triangle has an area of 52 square feet and a base of 13 feet. Are all triangles with these dimensions congruent?

SOLUTION

Using the formula from Example 2, the height is $h = \frac{2(52)}{13} = 8$ feet.

There are many triangles with these dimensions. Some are shown below.



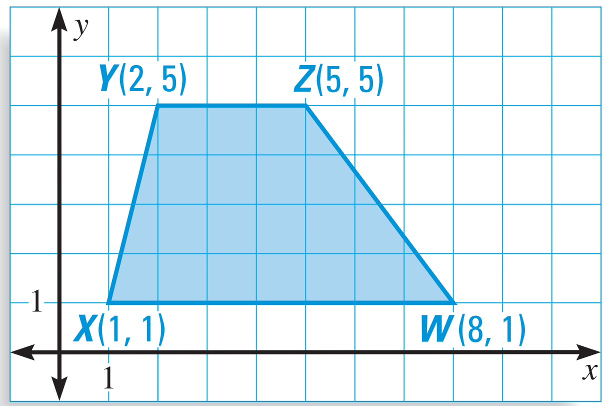
STUDENT HELP

Study Tip
Notice that the altitude of a triangle can be outside the triangle.

4. A rectangle has an area of 100 square meters and a height of 25 meters. Are all rectangles with these dimensions congruent?

EXAMPLE 4 *Finding the Area of a Trapezoid*

Find the area of trapezoid *WXYZ*.



SOLUTION

The height of *WXYZ* is $h = 5 - 1 = 4$.

Find the lengths of the bases.

$b_1 = YZ = 5 - 2 = 3$

$b_2 = XW = 8 - 1 = 7$

Substitute **4** for h, **3** for b_1, and **7** for b_2 to find the area of the trapezoid.

$A = \frac{1}{2}h(b_1 + b_2)$ **Formula for area of a trapezoid**

$= \frac{1}{2}(4)(3 + 7)$ **Substitute.**

$= 20$ **Simplify.**

▶ The area of trapezoid *WXYZ* is 20 square units.

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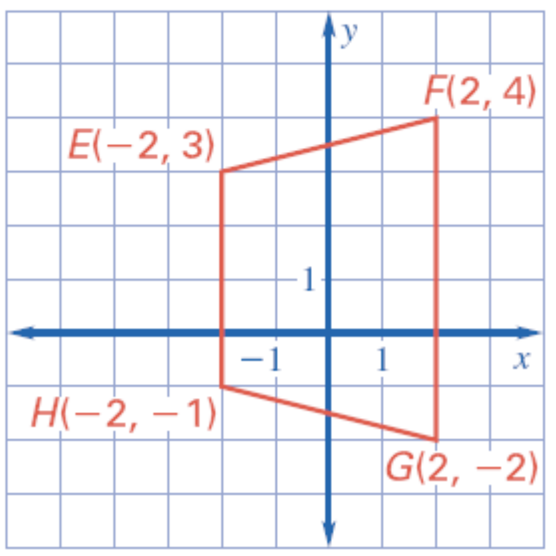
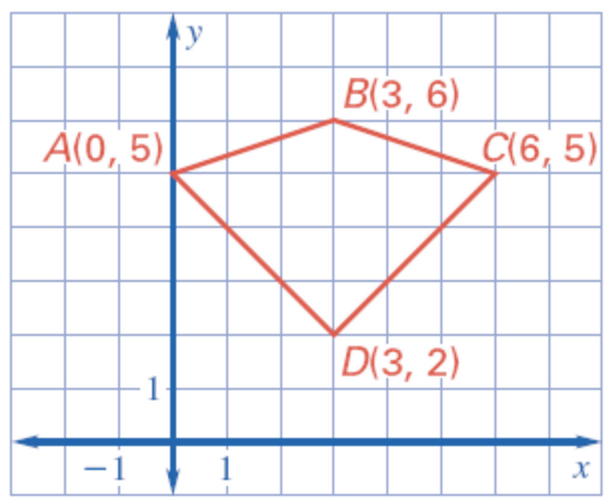
5. Find the area of trapezoid *EFGH*.

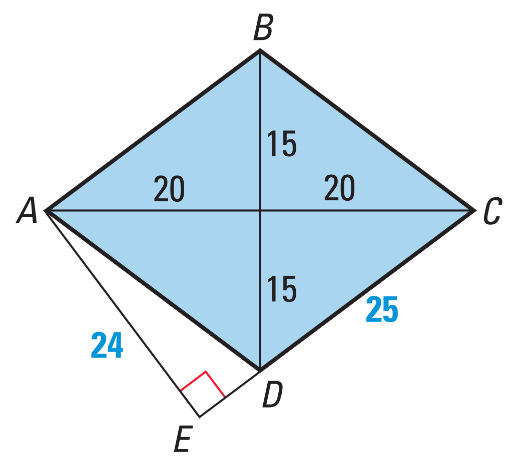


7. Find the area of kite *ABCD*.



EXAMPLE 5 *Finding the Area of a Rhombus*

Use the information given in the diagram to find the area of rhombus *ABCD*.



SOLUTION

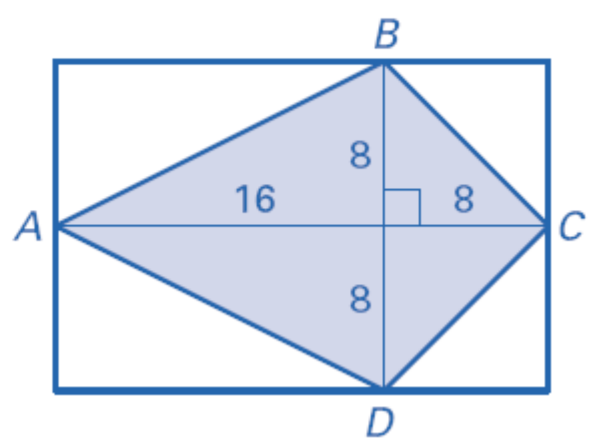
Method 1 Use the formula for the area of a rhombus. $d_1 = BD = 30$ and $d_2 = AC = 40$.

$$A = \frac{1}{2}d_1 d_2$$
$$= \frac{1}{2}(30)(40)$$
$$= 600 \text{ square units}$$

Method 2 Use the formula for the area of a parallelogram. $b = 25$ and $h = 24$.

$$A = bh = 25(24) = 600 \text{ square units}$$

8. Use the information given in the diagram to find the area of kite *ABCD*.



9. Find the area of rhombus *EFGH* if EG = 10 & FH = 15.

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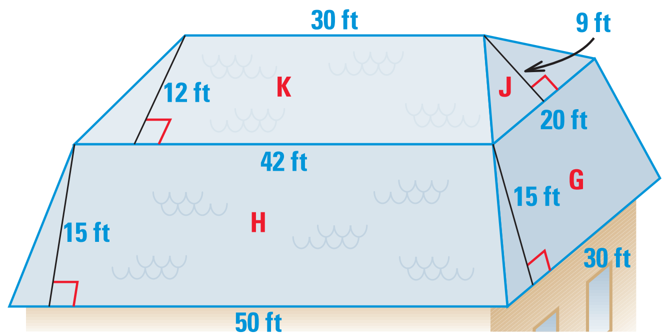
EXAMPLE 6 *Finding Areas*

REAL LIFE

ROOF Find the area of the roof. G, H, and K are trapezoids and J is a triangle. The hidden back and left sides of the roof are the same as the front and right sides.



STUDENT HELP

Study Tip
To check that the answer is reasonable, approximate each trapezoid by a rectangle. The area of H should be less than $50 \cdot 15$, but more than $40 \cdot 15$.

SOLUTION

Area of $J = \frac{1}{2}(20)(9) = 90 \text{ ft}^2$

Area of $H = \frac{1}{2}(15)(42 + 50) = 690 \text{ ft}^2$

Area of $G = \frac{1}{2}(15)(20 + 30) = 375 \text{ ft}^2$

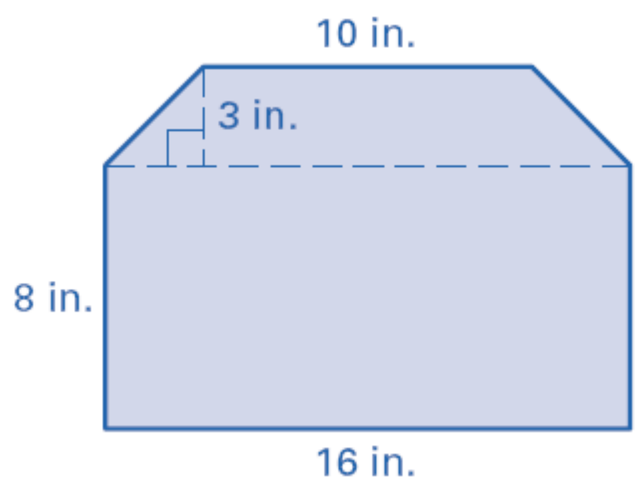
Area of $K = \frac{1}{2}(12)(30 + 42) = 432 \text{ ft}^2$

The roof has two congruent faces of each type.

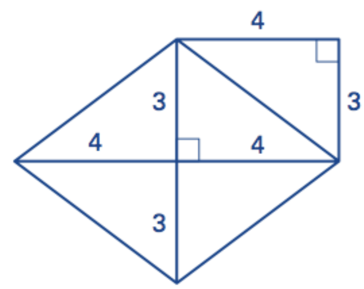
Total Area $= 2(90 + 375 + 690 + 432) = 3174$

The total area of the roof is 3174 square feet.

10. Example: The tray is designed to save space on cafeteria tables. How much table area does the tray use?



11. Find the area of the figure shown.



12. Describe two ways to find the area of a rhombus.

13. _____ Find the base length of a triangle with an area of 96 cm^2 and a height of 12 cm.

A. 8 cm
B. 12 cm
C. 16 cm
D. 32 cm

14. If you use AB as the base to find the area of $\square ABCD$, what should you use as the height?

Match the region with a formula for its area. Use each formula exactly once.

_____	Region 1		**A.** $A = s^2$
_____	Region 2		**B.** $A = \frac{1}{2}d_1d_2$
_____	Region 3		**C.** $A = \frac{1}{2}bh$
_____	Region 4		**D.** $A = \frac{1}{2}h(b_1 + b_2)$
_____	Region 5		**E.** A = bh